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Einstein's General Theory of Relativity at the Writing Table

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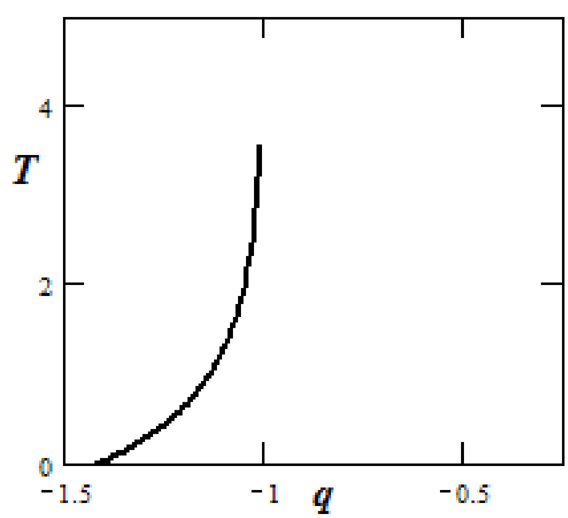
Abstract

This article is devoted to describing the possibility of substantiating A. Einstein's general theory of relativity by calculating, using its methods, the sums of some divergent series. For this, the series representing the Riemann *zeta* function is taken. The values of *zeta (-1)* and *zeta (0)* were calculated. The key element of the calculation is the singularity of the metric that occurs when solving Einstein's equations, describing the motion of a material particle simulating the calculation process. From the one hand, this once again makes it possible to make sure that singularities are a necessary element of the theory. From the other hand, it demonstrates that the calculation of the divergent series, which is non-computable on the conventional computer, can be performed on the relativistic one. At last, it proves that calculation itself can be regarded as a factor which influences the metric of the surrounding space i.e. the metric of the numerical axis in this case. The last wipes the difference between calculation and motion.



Traveling time of the particle in the system of the resting observer versus coordinate

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Keys: general theory of relativity, Riemann zeta function, divergent series, metric, singularity.

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1 Introduction

Einstein's general theory of relativity [1, 2] (GR) is the cornerstone for many areas of modern science: astrophysics, theory of elementary particles, field theory, etc. It can be said that it defines the basis of the modern physical picture of the world, laying the post-Newtonian concept of space and time, replacing the Newtonian theory of gravity. In other words, GR is a key element of modern knowledge in general. Therefore, special attention is paid to its experimental confirmation.

The first confirmations were received immediately after its creation. We are talking about calculating the anomalous value of the secular displacement of the perihelion of Mercury [2] and observing the deflection of the light beam in the field of a massive body [3]. Since then, many other observations and experiments have confirmed a significant amount of theoretical predictions, including gravitational time dilation, gravitational redshift, signal delay in a gravitational field, and gravitational radiation [4].

Despite this, at first glance, an exhaustive experimental confirmation, the question of the truth of GR is not closed. The reason is not so much in the lack of accuracy of measuring devices or in an insufficient number of experiments, but in the very concept of "experimental justification". Although the positive result of once more experiment in addition to the already existing ones adds confidence in the theory, in this case, GR, the question of its truth or falsity will thus never be closed and its acceptance or rejection will be for the most part a matter of personal preference. The same is true of experiments, allegedly refuting GR, like any other theory, although the role of negative experiments is much more significant.

This is the reason for the emergence of many alternative theories of gravity [5]. The motivations for creating them are, on the one hand, the desire to include in the theory of new observable phenomena, and, on the other hand, to rid it of elements that contradict the usual notions - singularities.

From a philosophical point of view, a singularity is the actualization of infinity, the penetration of which into theory has always been opposed by scientific thought. With the actual infinity of the opposite kind - Newtonian fluxions, science has coped with the concept of limits [6]. Nothing like this happens with singularities. Moreover, the presence of singularities in GR has been proved by various methods that differ from direct calculations [7].

This paper proposes a different approach to proving the validity of GR, in which singularities play a central role. It is about calculating the sums of some series, which in the usual sense, i.e. in a flat Euclidean metric diverge, while their calculation in a non-Euclidean metric, also singular, leads to a known final result. In this paper, we study the series for the Riemann zeta-function [8]. The main goal of the sum calculation problem is to find the metric ensuring the convergence of the series, which is achieved by solving the Einstein equation with a suitable source. In our case, the series is summed up in the moving reference frame, and the sum is calculated ("observed") at a resting frame. After finding the metric, the relativistic equation of motion of the material point moving in the vicinity of the singularity is solved. The sum of a series is defined as the distance traveled by the point from the initial position to the hitting of the singularity. When approaching the singularity, the "time" of calculation according to the clock of the stationary observer tends to infinity, which means that the sum of the divergent series is not computable in the ordinary, i.e. flat metric. In this sense, the computation process resembles the work of a relativistic supercomputer [9]. The accuracy of the calculation depends on the exact or approximate solution of Einstein equation. In some cases, it is possible to achieve absolute accuracy of the result, unattainable for any natural experiment.

The results of this work show that the confirmation of the Einstein's general theory of relativity can be found not only in the world of high energies and in the deep Cosmos but and at a simple writing table, what emphasizes its all-pervading nature.

2 Calculation the sum of a series for zeta (-1)

Riemann *zeta* function in the complex plane, i.e. for complex w is presented by the series

$$\zeta(w) = \sum_{n=1}^{\infty} n^{-w}, w = u + iv \quad (1)$$

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(u and v are real) which converges for $u>1$ [8]. For the values of the argument $u<1$, the series (1) diverges and here the values of the zeta function are determined using the analytical continuation [8]. Some calculations required the development of special methods. For example, *zeta (-1)* = -1/12 was first computed by Ramanujan using the summation method he specially developed.

For the finding *zeta* (-1) let us formulate a simple physical task - to calculate the distance which is gone by a particle for a time t, moving along a straight line with constant acceleration $a = 1$, the initial speed $v_0 = ½$ and the initial position $S_0 = 0$.

Mathematically, the answer can be represented as a finite sum

$$S(t) = \sum_{n=1}^{t} n = \frac{t(t+1)}{2} = S_0 + \frac{t^2}{2} + \frac{t}{2},$$
$$S_0 = S(0) = 0 \qquad (2)$$

If we assume the time t discrete and dimensionless the $S(t)$ represents the partial sums of (1).

Recall that the uniformly accelerated motion of the particle in classical physics requires according to Newton's second law, a constant force which operates on the particle in the direction of its movement. This effect can be achieved by placing at the point $x = 0$ (the direction of movement of the particle is taken as the axis OX of our frame of reference) an infinite plane coincident with the plane YOZ, having a constant mass density σ. The gravitational potential of this plane is equal to $\varphi(x)=2\pi\sigma Kx$, and the force acting on a unit point mass is equal to $E=-\varphi_x=-2\pi\sigma K$ and is directed along OX, K – is the gravitational constant. The expression for the space-time metric may be found from solving the Einstein equations [10]

$$R_{ik} - \frac{1}{2} g_{ik} R = \frac{8\pi K}{c^4} T_{ik} \qquad (3)$$

Here $R = R^i_i$ – is the trace of the Ricci tensor R^i_k, g_{ik} – is the metric tensor; T_{ik} - is the energy-momentum tensor; c – is the speed of light in vacuum; indices i, k have values 0, 1, 2, 3. Let us write the expression for the interval

$$ds^2 = g_{00}c^2dt^2 + g_{11}dx^2 + g_{22}dy^2 + g_{33}dz^2$$
$$x^0 = ct, x^1 = x, x^2 = y, x^3 = z$$
$$g_{00} = e^{v}, g_{11} = -e^{\lambda}, g_{22} = g_{33} = -1 \qquad (4)$$

The standard notations for g_{00} and g_{11} are used [10]. The solution is very similar to the Schwarzschild solution of the problem of finding the metric near a point mass [10]. The non-zero Christoffel symbols are follows[10]:

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$$\Gamma_{kl}^{i} = \frac{1}{2} g^{im} \left(\frac{\partial g_{mk}}{\partial x^l} + \frac{\partial g_{ml}}{\partial x^k} - \frac{\partial g_{kl}}{\partial x^i} \right)$$

$$\Gamma_{00}^{0} = \frac{\dot{\nu}}{2}, \Gamma_{00}^{1} = \frac{\nu'}{2} e^{\nu-\lambda}, \Gamma_{10}^{0} = \frac{\nu'}{2},$$

$$\Gamma_{10}^{1} = \frac{\dot{\lambda}}{2}, \Gamma_{11}^{0} = \frac{\dot{\lambda}}{2} e^{\lambda-\nu}, \Gamma_{11}^{1} = \frac{\lambda'}{2} \tag{5}$$

The point means a derivative on ct, prime - on $x^1=x$. For $x \neq 0$ where $T_{ik} = 0$ the equations (3) can be reduced to the equations $R_{ik}=0$, which for R_{00} and R_{11} lead to a single equation

$$\left[\frac{\nu''}{2} + \frac{\nu'}{2} \left(\frac{\nu'}{2} - \frac{\lambda'}{2} \right) \right] e^{\nu-\lambda} - \left[\frac{\ddot{\lambda}}{2} + \frac{\dot{\lambda}}{2} \left(\frac{\dot{\lambda}}{2} - \frac{\dot{\nu}}{2} \right) \right] = 0 \tag{6}$$

and the equation for R_{01} is reduced to identity. Assuming $\lambda = -\nu$, and all time derivatives equal zero, the last equation reduces to the form

$$\nu'' + (\nu')^2 = 0 \tag{7}$$

which has a solution $e^{\nu} = C_1 x + C_2$, $C_{1,2}$ – are constants. Their appropriate choice gives the desired solution

$$e^{\nu} = 1 + \frac{4\pi\sigma K}{c^2} x \tag{8}$$

considering the connection in the weak gravitational field limit of the component g_{00} and the Newtonian potential φ[10]: $g_{00}=1+2\varphi/c^2$. Let write the final form of the interval

$$ds^2 = \left(1 + \frac{4\pi\sigma K}{c^2} x \right) c^2 dt^2 - \left(1 + \frac{4\pi\sigma K}{c^2} x \right)^{-1} dx^2 - dy^2 - dz^2 \tag{9}$$

The expression for the interval (9) for $|x| << x_c$, where $x_c=c^2/4\pi\sigma K$, coincides with the well-known expression derived from the Einstein equivalence principle [11].

Let us treat the solutions of the equations of motion of the particle in question [10]

$$\frac{d^2x^i}{ds^2}+\Gamma^i_{kl}\frac{dx^k}{ds}\frac{dx^l}{ds}=0 \tag{10}$$

Given the nonzero Christoffel symbols and the expression for the derivative $\nu' = e^{\nu}/x_c$, we receive the system of two equations

$$2\frac{d^2x}{ds^2}-\frac{1}{x+x_c}\left(\frac{dx}{ds}\right)^2+\frac{x+x_c}{x_c^2}\left(\frac{dx^0}{ds}\right)^2=0$$
$$\frac{d^2x^0}{ds^2}+\frac{1}{x+x_c}\frac{dx^0}{ds}\frac{dx}{ds}=0 \tag{11}$$

The second equation in (11) is integrated and gives

$$\left(\frac{dx^0}{ds}\right)^2=\frac{B}{\left(1+\frac{x}{x_c}\right)^2} \tag{12}$$

B is a constant. After substitution (12) into the first equation in (11) it looks as follows

$$2\frac{d^2x}{ds^2}-\frac{1}{x+x_c}\left(\frac{dx}{ds}\right)^2+\frac{B}{x+x_c}=0 \tag{13}$$

Equation (13) can be integrated once that leads to an expression for the 4-speed (D - is a constant)

$$\left(\frac{dx}{ds}\right)^2=B+D\left(1+\frac{x}{x_c}\right) \tag{14}$$

or in another form in variable x^0

$$\left(\frac{dx}{dx^0}\right)^2=\left(1+\frac{x}{x_c}\right)^2+\frac{D}{B}\left(1+\frac{x}{x_c}\right)^3 \tag{15}$$

The constants of integration can be found comparing the expression of (15) with a corresponding classical result. A classic case corresponds to $|\ x\ | << x_c$. Expanding the right side of the (15) in x we receive the classical limit of (15)

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$$v^2(x) = \left(\frac{dx}{cdt}\right)^2 = 1 + \frac{D}{B} + \left(2 + \frac{3D}{B}\right)\frac{x}{x_c} \qquad (16)$$

It should be compared with the classical result following from the law of conservation of energy

$$v^2(x) = \left(\frac{dx}{dt}\right)^2 = \pm\frac{c^2}{x_c}(x_0 - x) \qquad (17)$$

x_0 – is the value of x wherein the speed $v = 0$, signs "±" ensure the positivity of the right side, taking into account that $|x_0| \geq |x|$. So for the sign "-" we receive

$$\frac{D}{B} = -1 \qquad (18)$$

Approach the massive particle to the point $x = -x_c$ according to the formula (15) spends an infinite time from the point of view of a resting observer, as is seen in Fig. 1:

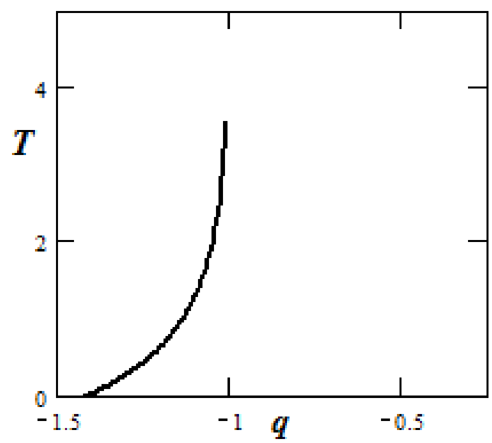

Fig. 1. | Traveling time T = ct/xc of the particle in the system of the resting observer received from the solution of (15) versus coordinate q = x/xc.

If we'll try to calculate the sum *S(∞)=zeta (-1)* (2) with the help of formulas derived above we'll find that the condition for the velocity $v_0 = 0{,}5$ cannot be satisfied for real x as seen from the Fig. 2.

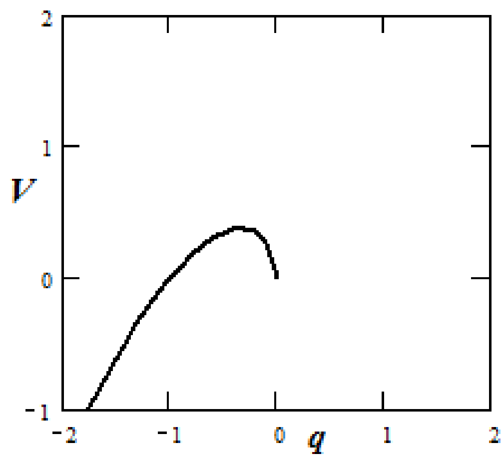

Fig. 2 | Velocity of the particle (18) in the system of the resting observer versus coordinate; $V(q)=(1+q)(-q)^{1/2}$, $q=x/x_c$. Point q_0 = -1.41965 corresponds to the speed v_0 = -0,5.

Thus we slightly change our task and instead of searching of the sum *S(∞)* (1) we will find another sum *S*1*(∞)*, where

$$S1(t)=\sum_1^t (n-1)=S1_0+\frac{t^2}{2}-\frac{t}{2}$$
$$S1_0=0 \qquad (19)$$

It is obvious that *S(∞)=S*1*(∞)* +ζ(0)=*S*1*(∞)*-0,5 [8]. The initial values for the *S*1 are just the same as for the *S* except for the value for the initial speed which looks as follows: v_0 = -0.5, and easily can be satisfied what gives the value for the q_0=-1.41965.

The condition $S1_0$ = 0, means that the initial point should be placed at the point q_0 = -1.41965. Then the distance which has been traveled by a massive particle before its stop at point $q_c = x/x_c$ = - 1 is equal to -1 – (-1.41965) - 0,5 = -0.08035 from the point of view of the distant observer. This coincides with the exact value for ζ(-1)=-0,08333 with relative error 3.576 % what is the consequences of using the approximate formula (9) for the metric.

3 Calculation the sum of a series for zeta (0)

To calculate *zeta* (0), notice that corresponding partial sums $S(t)=\sum_{n=1}^{t} 1 = t$ look as expressions for a distance traveled by the material particle moving with constant velocity v = 1 i.e. without acting any force. Then the calculation algorithm should be changed by placing two planes perpendicular to the *X*-axis and identical to that considered above. Doing similar calculations, we get an expression for the interval *ds* (terms $\sim dy^2$ and $\sim dz^2$ we omit)

$$ds^2=\left(1+\frac{2x}{x_c}\right)c^2dt^2-\left(1+\frac{2x}{x_c}\right)^{-1}dx^2, x<-\frac{x_c}{2}$$
$$ds^2=\left(1-\frac{2x}{x_c}\right)c^2dt^2-\left(1-\frac{2x}{x_c}\right)^{-1}dx^2, x>\frac{x_c}{2}$$
$$ds^2=c^2dt^2-dx^2, -\frac{x_c}{2}<x<\frac{x_c}{2} \qquad (20)$$

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So the value *zeta*(0) = $S(\infty)$ when we use the last expression for metric (20) taking into account that the space of uniform motion is bounded by singularities. Solving the equations of motion (10) for $\Gamma_{kl} = 0$ we find: $x - x_0 = s$, $t - t_0 = s$ (in dimensionless form; x_0, t_0 are constants of integrating, s is a proper time) and choosing $x_0 = 1$ and $t_0 = 0$ we have $x - 1 = t = S(t)$. From the first equation we find that the particle moving to the right crosses the horizon (fall down in the singularity $x = x_c/2$) having the magnitude of $S_h = S(-0.5) = -0.5$. Because x can't exceed further as well as $S(t)$ then we conclude that

$$\zeta(0) = S(\infty) = S_h = -0.5 \tag{21}$$

what coincides with the right value [8].

4. Discussion

When calculating the values of the *zeta*-function, were guided by the well-known position of GR — after a particle hits the singularity, it becomes inaccessible for a resting observer. From the point of view of calculation, this means that the desired sum of the series stops changing and acquires its final value.

Let us dwell on some details of the calculations.

Expression (9) for the metric is an approximate solution of the problem since it corresponds to the motion of a particle under the action of a constant force E, which is known to be a Lorentz invariant [12]. Our task was to find the metric corresponding to the particle motion with constant acceleration (in the resting observer system w = *const.* It is known that acceleration w' in the reference system moving with velocity v and the acceleration w in the resting one are connected by the formula [12]

$$w = \left(1 - \frac{v^2}{c^2}\right)^{3/2} w' \tag{22}$$

Thus to apply the proposed method for this solution, it is necessary to assume that the mass density σ is not constant due to alternating $v(x)$. However, the Einstein equations in this case are much more complicated and are unlikely to have a closed solution. Therefore, as well as in [13], we will use the expression (9) for the metric, especially since the error, as it turns out, will be small. Really, it follows from Fig. 2 that maximal value of the quotient $(v/c)^2 \approx 0.14$ what confirms the above consideration. Exactly the same considerations allow us to explain the accuracy of the calculation result for *zeta*(0) (21) since in this case, the velocity v is constant.

The behavior of a material point moving in a metric (9) resembles a movement in a gravitational field of a black hole, more precisely of its one-dimensional analog. However, the calculation for *zeta*(-1) of the distance traveled between the singularity and the position on the real axis, where the velocity of the point is known, is performed for the so-called, white hole, which is a temporary reflection of the black hole. This is explained by the choice of the sign for the potential $\varphi(x)$ and will not lead to the need to change the calculations since the equations are symmetric with respect to the reflection in time.

The results presented in this paper allow us to put an end to the ongoing attempts to eliminate singularities from A. Einstein's general theory of relativity. Such an approach to solving scientific disagreements has historical roots. Recall that the first confirmations of Lobachevsky's geometry were related with the calculation of some integrals [14].

The question of how the results of this work will change if we take into account the cosmological term in equations (3) is worthy of interest. On the one hand, this will lead to a considerable complication in obtaining both solutions of the Einstein equations and relativistic equations of motion. On the other hand, it can improve the accuracy of calculating zeta(-1). Then it would allow speaking about the close connection between computations and Cosmos and would be an argument in favor of the Pythagorean view of the nature of numbers. In any case, this is the subject of separate publications.

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5. Conclusion

This article is devoted to describing the possibility of substantiating A. Einstein's general theory of relativity by calculating, using its methods, the sums of some divergent series, for which the series representing the Riemann *zeta* function are taken. The calculation algorithm resembles the work of a relativistic supercomputer when the calculation is performed in a moving frame of reference, and the definition ("observation") of the result performs in a resting frame. The key element of the calculation is the singularity of the metric that occurs when solving Einstein's equations, describing the motion of a material particle simulating the calculation process.

The accuracy of the calculation depends on how accurately the metric is determined. For *zeta*(0) the result coincides with the exact one. This once again makes it possible to make sure that singularities are a necessary element of the theory and attempts to construct gravitational theories free from them are only a tribute to personal preferences.

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